EM polje

Lagranžijan

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad (1)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{i} \quad A_\mu = (A_0, -\vec{A}) \checkmark$$

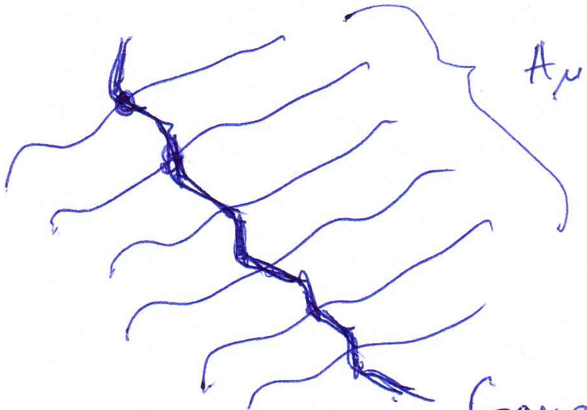
$$\vec{E} = -\vec{\nabla} A^0 - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A} \qquad \Pi^\mu = \frac{\partial \mathcal{L}}{\partial \dot{A}_\mu}$$

J-na kretanja - slobodno EM polje

$$(1) \Rightarrow \quad \Box A_\mu - \partial_\mu (\partial^\nu A_\nu) = 0 \qquad (2)$$

Potencijali definisani do na gauge transfor-macije

$$A_\mu(x) \rightarrow A'_\mu(x') = A_\mu(x) + \partial_\mu \Lambda(x)$$



$$A_\mu(x) + \partial_\mu \Lambda(x)$$

Gauge uslov (Baždarni uslov, uslov kalibracije)

Uklanja višak stepeni slobode

$$\begin{cases} \partial_\mu A^\mu = 0 & \text{Lorenz} \\ \partial_i A^i = 0 & \text{Coulo} \end{cases}$$

1 Pokazati da propagator za klasično EM polje NE postoji. Posledica toga je da se EM polje NE može direktno kvantovati

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial_\mu F^{\mu\nu} = j^\nu \longrightarrow$$ pogledati prvi par na poslednjoj strani

$$\rightarrow \quad \partial_\mu \left(\partial^\mu A^\lambda - \partial^\lambda A^\mu \right) = j^\lambda$$

$\partial_\mu A^\mu = \partial^\mu A_\mu$

$$\left(g^{\lambda\mu} \Box - \partial^\lambda \partial^\mu \right) A_\mu = j^\lambda$$

Odgovarajuća j-na za Green-ovu f-ju

$$\left(g^{\lambda\mu} \Box - \partial^\lambda \partial^\mu \right) D_{\mu\nu}(x-x') = g^\lambda{}_\nu \, \delta^4(x-x')$$

F-jeova transformacija prethodne f-ke daje

$$- \left(g^{\lambda\mu} k^2 - k^\lambda k^\mu \right) D_{\mu\nu}(k) = g^\lambda{}_\nu \qquad (1)$$

Dakle, $D_{\mu\nu}(k)$ mora biti oblika

$$D_{\mu\nu}(k) = a\, g_{\mu\nu} + b\, k_\mu k_\nu \qquad (2)$$

Hipoteza!

a, b – Лоренцови скалари

Враћајући (2) у (1)

$$-(g^{\lambda\mu}k^2 - k^\lambda k^\mu)(a g_{\mu\nu} + b k_\mu k_\nu) = g^\lambda{}_\nu$$

$$-(a g^{\lambda\mu} g_{\mu\nu} k^2 + b g^{\lambda\mu} k^2 k_\mu k_\nu - a g_{\mu\nu} k^\lambda k^\mu - b k^\lambda k^\mu k_\mu k_\nu) = g^\lambda{}_\nu$$

$$-(a g^\lambda{}_\nu k^2 + b g^{\lambda\mu} k^2 k_\mu k_\nu - a g_{\mu\nu} k^\lambda k^\mu - b k^\lambda k_\nu k^2) = g^\lambda{}_\nu$$

$$-(a g^\lambda{}_\nu k^2 + \cancel{b k^2 k^\lambda k_\nu} - a g_{\mu\nu} k^\lambda k^\mu - \cancel{b k^\lambda k_\nu k^2}) = g^\lambda{}_\nu$$

$$-a g^\lambda{}_\nu k^2 + a k^\lambda k_\nu = g^\lambda{}_\nu \qquad (3)$$

Дакле, одозго видимо да се b не може одредити. Из (3) се види да први члан каже да је $a = -\frac{1}{k^2}$, а други члан каже да је $a = 0$. Дакле, и a се не може одредити. Пропагатор не постоји.

$a = -\frac{1}{k^2}$ биће нула само ако је маса мировања бесконачна

Разлог за непостојање пропагатора:

$\vec{E}$ i $\vec{B}$ — 6 veličina

$$\vec{\nabla}\cdot\vec{E} = \rho \qquad \vec{\nabla}\times\vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j} \qquad (1)$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}} \qquad (2)$$

ograničenja (j-ne veze ukupno: 4)

6 veličina i 4 j-ne veze = dva nezavisna stepena slobode u EM polju.

$A_\mu \rightarrow$ 4 veličine (stepeni slobode) i nisu svi nezavisni

redundancija: potreban je dodatan uslov da se ukloni. Ispostavlja se da je to gauge uslov

2. Po uzoru na komutacione relacije za skalarno KG polje, izvesti komutacione relacije za EM polje u Coulombovom gauge-u.

Coulombov gauge uslov

$$\mathrm{div}\,\vec{A} = 0 \qquad (1)$$

U principu nas interesuje

$$[A_\mu, \Pi_\mu] = ?$$

Ali

$$(*) \quad \pi^0 = \frac{\partial L}{\partial(\partial_0 A_0)} = 0$$ (Mandl, Shaw)

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Zbog gornjeg[(*)], tražimo

$$[A^i(\vec{x},t), \pi^j(\vec{x}',t)] = i\delta_{ij}\,\delta^3(\vec{x}-\vec{x}') \quad (2)$$

Ali (2) nije konzistentno sa (1) jer

$$[\mathrm{div}\,\vec{A}(\vec{x},t), \pi^j(\vec{x}',t)] = i\delta_{ij}\,\partial_i\,\delta^3(\vec{x}-\vec{x}') \neq 0$$

$$\text{LS} \neq \text{DS}$$

Dakle, valja promeniti komutacione relacije. Zamenićemo δ_{ij} sa tenzorom ranga 2! Δ_{ij} i zapisati $\delta^3(\vec{x}-\vec{x}')$ u integralnoj formi

$$\left[A^i(\vec{x},t), \pi^j(\vec{x}',t)\right] = i\int \frac{d^3\vec{k}}{(2\pi)^3}\,\Delta_{ij}\, e^{-i\vec{k}(\vec{x}-\vec{x}')}$$

Promenjiviči (1)

$$\left[\mathrm{div}\vec{A}(\vec{x},t), \pi^j(\vec{x}',t)\right] = \frac{1}{(2\pi)^3}\int d^3\vec{k}\left(\sum_i k_i\,\Delta_{ij}\right) e^{-i\vec{k}(\vec{x}-\vec{x}')} \quad (3)$$

Uslov da D.S. od (3) bude nula je da važi jednakost

$$\Delta_{ij} = \delta_{ij} - \frac{k_i k_j}{|\vec{k}|^2} \quad (4)$$

jer $\sum_i k_i\Delta_{ij} = \sum_i k_i\left(\delta_{ij} - \frac{k_i k_j}{|\vec{k}|^2}\right) =$

$$= \sum_i k_i\delta_{ij} - \sum_i \frac{k_i^2 k_j}{|\vec{k}|^2} = k_j - k_j\frac{\sum_i k_i^2}{|\vec{k}|^2} = k_j - k_j = 0$$

Dakle, komutacione relacije u Coulombovom gaugu

$$\left[A^i(\vec{x},t), \pi^j(\vec{x}',t)\right] = i\,\Delta_{ij}\,\delta^3(\vec{x}-\vec{x}')$$

Naravno

$$\left[A^i(\vec{x},t), A^j(\vec{x}',t)\right] = 0 = \left[\pi^i(\vec{x},t), \pi^j(\vec{x}',t)\right]$$

3. Pokazati da Lorencov uslov odredjuje da $\overset{EM}{\text{v}}$ polje ima dva nezavisna stepena slobode. Koristiti j-ne kretanja EM polja.

Lorenz gauge uslov

$$\partial_\mu A^\mu = 0 \qquad (1)$$

J-ne kretanja $\qquad (A_\nu \equiv A_\nu(x))$

$$\Box A_\mu - \partial_\mu (\partial^\nu A_\nu) = 0 \qquad (2)$$

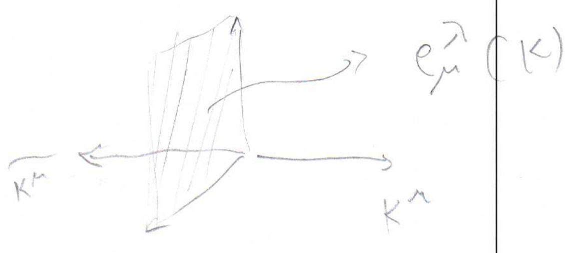
Furjeov transform od $A_\mu(x) = \int d^4k\, e^{ikx} A_\mu($

Zamenti ovo u (2); dobija se:

$$-k^2 A_\mu(k) + k_\mu (k^\nu A_\nu(k)) = 0 \qquad (3)$$

J-na kretanje u impulsnom prostoru

Razlozimo $A_\mu(k)$ preko 4 nezavisna kvadrivektora



$$k^\mu = (E, \vec{k})$$

$$\tilde{k}^\mu = (E, -\vec{k}) \qquad (4)$$

$$e^\lambda_\mu(k) \quad , \lambda = 1, 2$$

pri čemu je ispunjeno

$$k^\mu e^\lambda_\mu = 0 \qquad (5)$$

$\longleftarrow$ nametnut uslov!

Razlaganje od $A_\mu(K)$ glasi

$$A_\mu(K) = a_\lambda(K)\, e_\mu^\lambda + b(K)\, K_\mu + C(K)\, \tilde{K}_\mu \qquad (6)$$

Zamenivši (6) u (3) $\Rightarrow$

$$-K^2\left(a_\lambda(K)\, e_\mu^\lambda + b(K)\, K_\mu + C(K)\, \tilde{K}_\mu\right) +$$

$$K_\mu\left(\underbrace{K^\nu\left(a_\lambda(K)\, e_\nu^\lambda\right.}_{=0 \text{ zbog } (5)} + b(K)\, K_\nu + C(K)\, \tilde{K}_\nu\left.\right)\right) = 0$$

Dalje

$$\cancel{-K^2}\left(a_\lambda(K)\, e_\mu^\lambda + \cancel{b(K)\, K_\mu} + C(K)\, \tilde{K}_\mu\right) +$$

$$\cancel{K_\mu}\left(\cancel{b(K)\, K^2} + C(K)(K\cdot\tilde{K})\right) = 0$$

(*) $b(K)$ je neodređeno jednačinom kretanja i $\boxed{\text{bira se}}$ da bude nula

$$-K^2 a_\lambda(K)\, e_\mu^\lambda - K^2\, C(K)\, \tilde{K}_\mu + K_\mu\, C(K)\, (K\cdot\tilde{K}) = \qquad (7)$$

(7) će biti zadovoljeno samo ako je

$$K^2 a_\lambda(K) = C(K) = 0$$

v. pozadi

Ako se uzme $K^2 = 0$ (masa mikrotalasnog kvanta EM polja nula) $\Downarrow$ $a_\lambda(K) \neq 0$

onda je iz (6)

$$A_\mu(K) = a_\lambda(K)\, e_\mu^\lambda \qquad (8)$$

iz (8), a zbog (5)

$k^\mu A_\mu(k) = 0$, što je $\Leftrightarrow$ (1)

Dakle izbor da je $b(k) = 0$ je

$\Longleftrightarrow$ Lorenz gauge uslovu

EM $\leftrightarrow$ dva nezavisna stepena slobode (polarizacija)

FOTON – PROSTOR STANJA DVODIMENZIONALAN

$\searrow$ efektivni spin fotona $\frac{1}{2}$

4

Lagranžijan koji opisuje slobodne fermione i fotone glasi

$$\mathcal{L}_0 = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Gauge transformacije ovog Lagranžijana

$$A^\mu(x) \rightarrow A^\mu(x) - \frac{1}{e}\partial^\mu \Lambda(x) \quad \text{(EM polje)}$$

V. pozadi

$$\Psi(x) \rightarrow e^{i\Lambda(x)}\Psi(x) \quad \text{(Dirakovo polje)}$$

a) Naći minimalni interakcioni član $\mathcal{L}_{int}$ takav da je ukupni $\mathcal{L}$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$$

invarijantan na gauge transformacije

b) Napisati Lagranžijan u formi

$$\mathcal{L} = \bar{\Psi}(iD\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

i pokazati da se $D_\mu\Psi$ transformiše isto kao Ψ pod gauge transformacijama

---

$$A^\mu \rightarrow A^\mu - \frac{1}{e}\partial^\mu\Lambda$$

$F_{\mu\nu}$ po def. invarijantno (za domaći)

Mošićki

$$\psi \to e^{i\Lambda}\psi$$

$$\bar\psi \to \bar\psi e^{-i\Lambda}$$

$$\mathcal{L}_0 \to \bar\psi e^{-i\Lambda}(i\not\partial - m)e^{i\Lambda}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= i\bar\psi e^{-i\Lambda}\not\partial(e^{i\Lambda}\psi) - m\bar\psi\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= i\bar\psi e^{-i\Lambda}(i\not\partial\Lambda e^{i\Lambda}\psi + \underbrace{e^{i\Lambda}\not\partial\psi) - m\bar\psi\psi}_{\text{objediniti}} -$$

$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= -\bar\psi(\not\partial\Lambda)\psi + \bar\psi(i\not\partial - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= \mathcal{L}_0 - \bar\psi(\not\partial\Lambda)\psi$$

$$= \mathcal{L}_0 - \bar\psi(\gamma^\mu\partial_\mu\Lambda)\psi$$

$$= \mathcal{L}_0 + e\bar\psi(\gamma^\mu\delta A_\mu)\psi$$

ОБЕ ПРОМЕНЕ СЕ ПОТИРУ

DAKLE

$$\mathcal{L}'_{int} \to \mathcal{L}_{int} - e\bar\psi\gamma^\mu\delta A_\mu\psi$$

odnosno

$$\mathcal{L}_{int} = -e\bar\psi\gamma^\mu A_\mu\psi = -eA_\mu\underbrace{\bar\psi\gamma^\mu\psi}_{\text{DIRAKOVO POLJE}}$$

$$= -eA_\mu j^\mu$$

b) $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$

$$= \bar{\Psi}(i\not{\partial} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - eA_\mu\bar{\Psi}\gamma^\mu\Psi$$

$$= \bar{\Psi}(i\gamma^\mu(\partial_\mu + ieA_\mu) - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= \bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$D_\mu = \partial_\mu + ieA_\mu \rightarrow$ kovarijantni izvod (minimalna preskripcija)

$$D_\mu\Psi = \partial_\mu\Psi + ieA_\mu\Psi$$

$$\rightarrow \partial_\mu(e^{i\Lambda}\Psi) + ie(A_\mu - \frac{1}{e}\partial_\mu\Lambda)e^{i\Lambda}\Psi$$

$$= \cancel{i(\partial_\mu\Lambda)e^{i\Lambda}\Psi} + e^{i\Lambda}\partial_\mu\Psi$$

$$+ ieA_\mu e^{i\Lambda}\Psi \cancel{- i(\partial_\mu\Lambda)e^{i\Lambda}\Psi}$$

$$= e^{i\Lambda}(\partial_\mu\Psi + ieA_\mu\Psi) = e^{i\Lambda}D_\mu\Psi$$

I zaista, transformiše se kao

Ψ $\left(\Psi \rightarrow e^{i\Lambda(x)}\Psi\right)$

Fajnmanova pravila za kvantnu elektrodinamiku

1. Za svaki verteks (tačku) čvor upisati $ie\gamma^\alpha$

2. Fotonski propagator (unutrašnja linija)

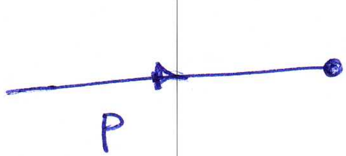
$$iD_{F\alpha\beta}(k) = i\frac{-g_{\alpha\beta}}{k^2+i\varepsilon}$$

3. Leptonski propagator (unutrašnja linija)

$$iS_F(p) = \frac{i}{\not p - m + i\varepsilon}$$

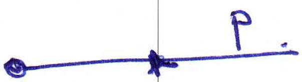
4. Spoljašnje linije

Dolazni elektron



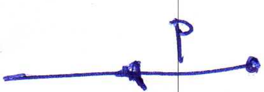
$u_r(\vec p)$

odlazni elektron



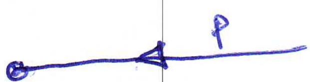
$\bar u_r(\vec p)$

Dolazni pozitron



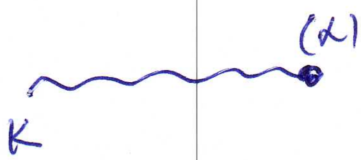
$\bar v_r(\vec p)$

Odlazni pozitron



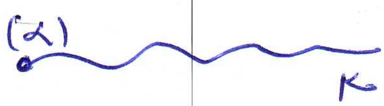
$v_r(\vec p)$

Dolazni foton



$\varepsilon_{r\alpha}(\vec k)$

odlazni foton

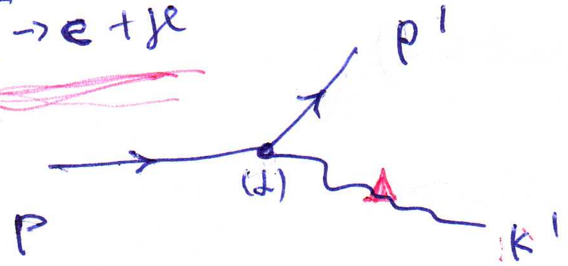


$\varepsilon^*_{r\alpha}(\vec k)$

$r=1,2 \to$ prebrojava spinove i polarizacije!

5. Spinorski faktori (γ matrice, S_F i spinori $u_r, v_r, \bar{u}_r, \bar{v}_r$) se pišu sa desna na levo sledeći strelice na fermionskim linijama

Primer: $e^- \to e^- + \gamma$



$\vec{k}' = \vec{p} - \vec{p}'$

$$\mathcal{M} = ie\, \bar{u}(\vec{p}')\, \gamma^{\alpha} \varepsilon^{*}_{r\alpha}(\vec{k}')\, u(\vec{p})$$

$$= ie\, \bar{u}(\vec{p}')\, \not{\varepsilon}^{*}(\vec{k}')\, u(\vec{p})$$

6. Za svaku zatvorenu fermionsku petlju, uzima se $\underline{tr}$ izraza i množi sa (-1)

7. Za svaki kvadrivektor impulsa q koji nije fiksiran zakonima održanja energije i impulsa vrši se integracija $\frac{1}{(2\pi)^4}\int d^4 q$

8. Ceo izraz za amplitudu se množi sa $(+1)$ ili (-1) ako je potreban paran ili neparan broj izmena da bi se raspored fermionskih operatora doveo u normalno uređenje (npr: $a_1^{+} a_2^{+} a_2 a_1$)

↳ ili na drugi način rečeno →

$$\boxed{\hat{\vec{P}} = \hat{\vec{p}} - \frac{q}{c}\vec{A}}$$

Координатна репрезентација

$$-iD_j = -i\partial_j - qA_j$$

$$\Downarrow$$

$$\boxed{D_j = \partial_j + iqA_j}$$

Нерелативистички

$$P_\mu = (E, -\vec{p})$$

$$p_\mu \Rightarrow (i\partial_t, i\partial_j) = i(\partial_t, \partial_j) = i\partial_\mu$$

$$\boxed{P_\mu = p_\mu - \frac{q}{c}A_\mu}$$

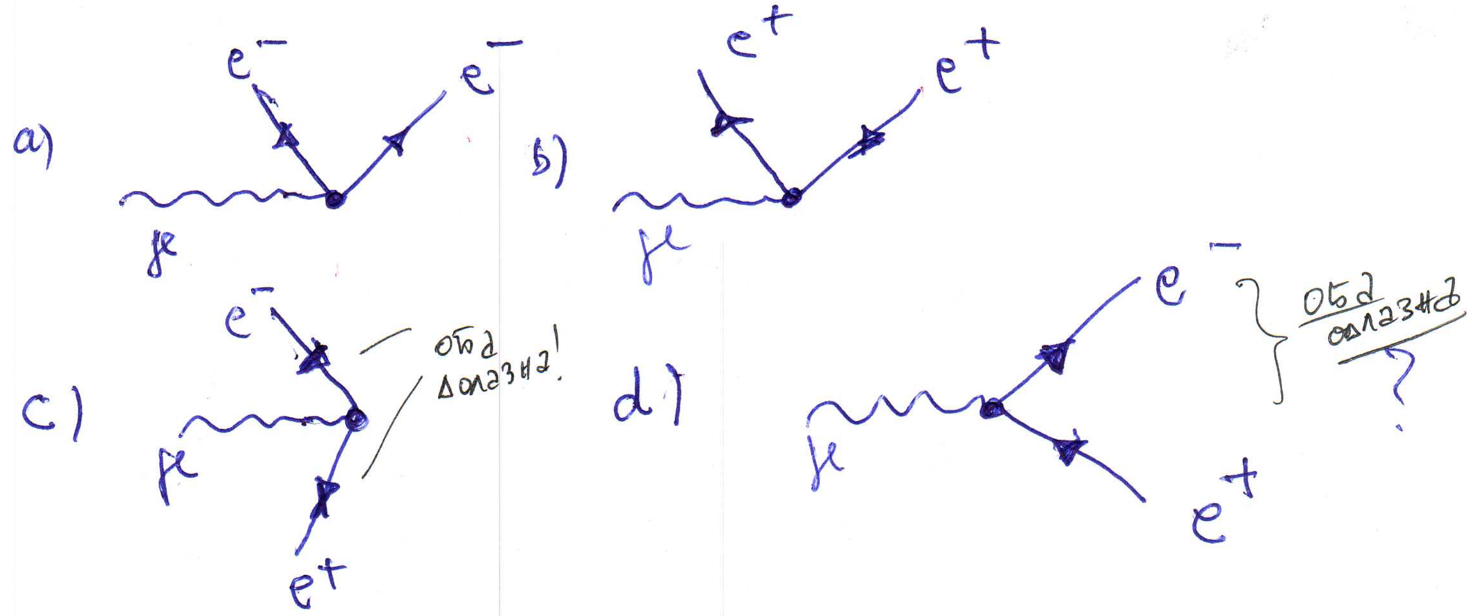
$$iD_\mu = i\partial_\mu - \frac{q}{c}A_\mu = i\left(\partial_\mu + i\frac{q}{c}A_\mu\right), \quad c = 1$$

$$= i(\partial_\mu + iqA_\mu)$$

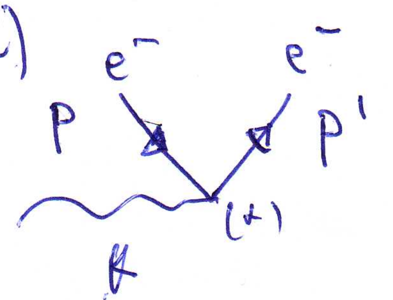
$$\boxed{D_\mu = \partial_\mu + iqA_\mu}$$

Релативистички

1. Kako glase Fajnmanove amplitude za sledeće dijagrame:



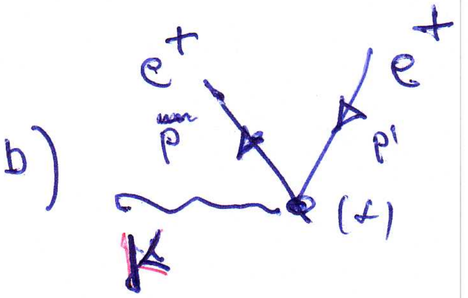
Komentar: Ovo gore su apsorpcije fotona (dolazni fotoni)



$$\mathcal{M} = ie\, \bar{u}_r(\vec{p}\,')\gamma^\alpha \varepsilon_\alpha(\vec{k})\, u_r(\vec{p})$$

$u_r(\vec{p}) = u(\vec{p})$ itd!

Ako nema potrebe da se sumira po r-ovima



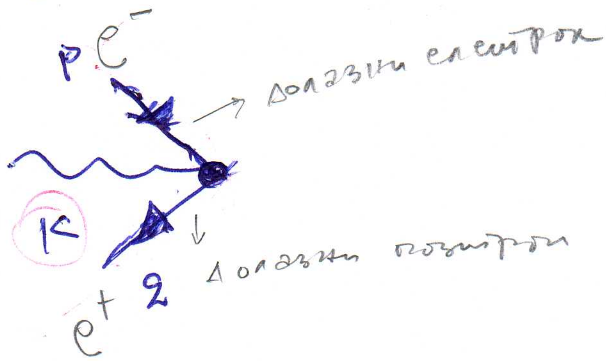
$$\mathcal{M} = \cancel{ie\, v(\vec{p})\gamma^\alpha \varepsilon_\alpha(\vec{k})\, \bar{v}(\vec{p}\,')}$$

$$\mathcal{M} = ie\, \bar{v}_r(\vec{p})\gamma^\alpha \varepsilon_\alpha(\vec{k})\, v_r(\vec{p}\,')$$

c)

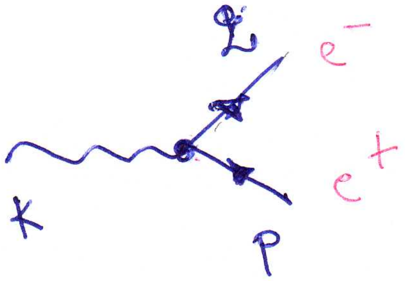
$$\mathcal{M} = ie\, \bar{v}(\vec{q})\, \gamma^{\alpha} \varepsilon_{\alpha}(\vec{k})\, u(\vec{p})$$

$$= ie\, \bar{v}(\vec{q})\, \not{\varepsilon}(\vec{k})\, u(\vec{p})$$

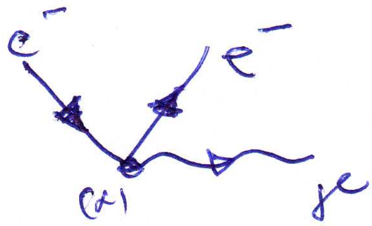


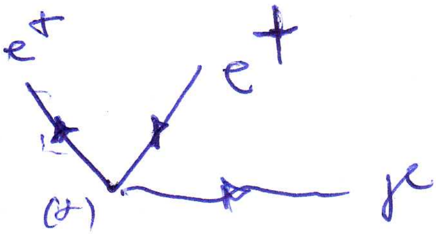
d)

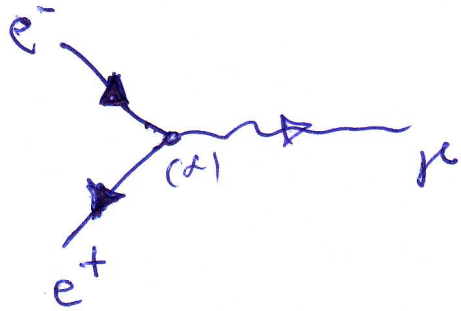
$$\mathcal{M} = ie\, \bar{u}(\vec{q})\, \not{\varepsilon}(\vec{k})\, v(\vec{p})$$

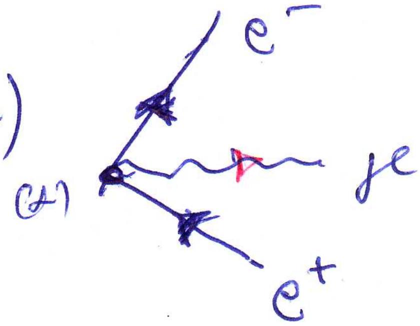


2. Kako glase Feynmanove amplitude za sledeće dijagrame:

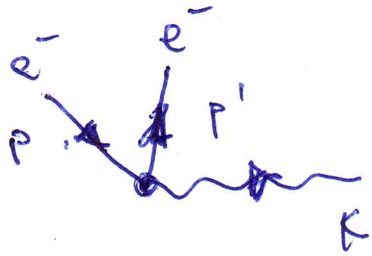
a) 

b) 

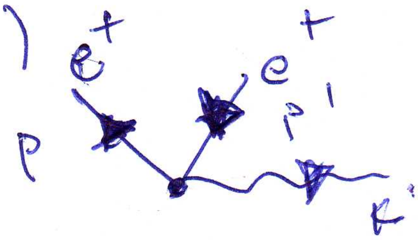
c) 

d)  ?

$\boxed{\text{Komentar: Ovo gore su emisije fotona}}$

a) 

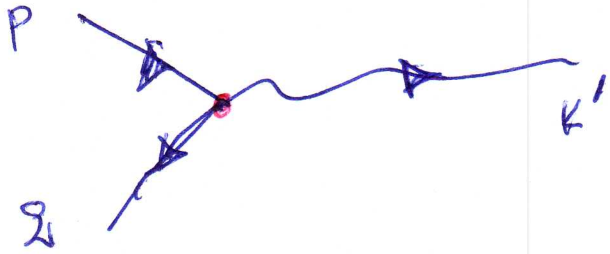
$$\mathcal{M} = ie\ \bar{u}(\vec{p}\,')\,\not{\epsilon}(\vec{k})\,u(\vec{p})$$

b) 

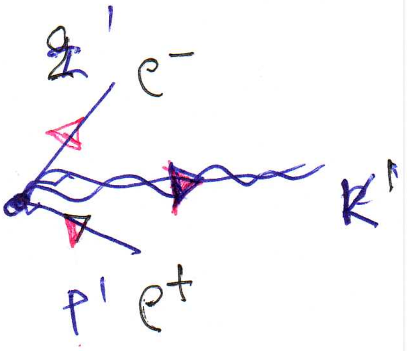
$$\mathcal{M} = \cancel{ie\ \bar{v}(\vec{p}\,')\,\not{\epsilon}(\vec{k})\ v(\vec{p})}$$

$$= ie\ \bar{v}(\vec{p})\,\not{\epsilon}(\vec{k})\,v(\vec{p}\,')$$

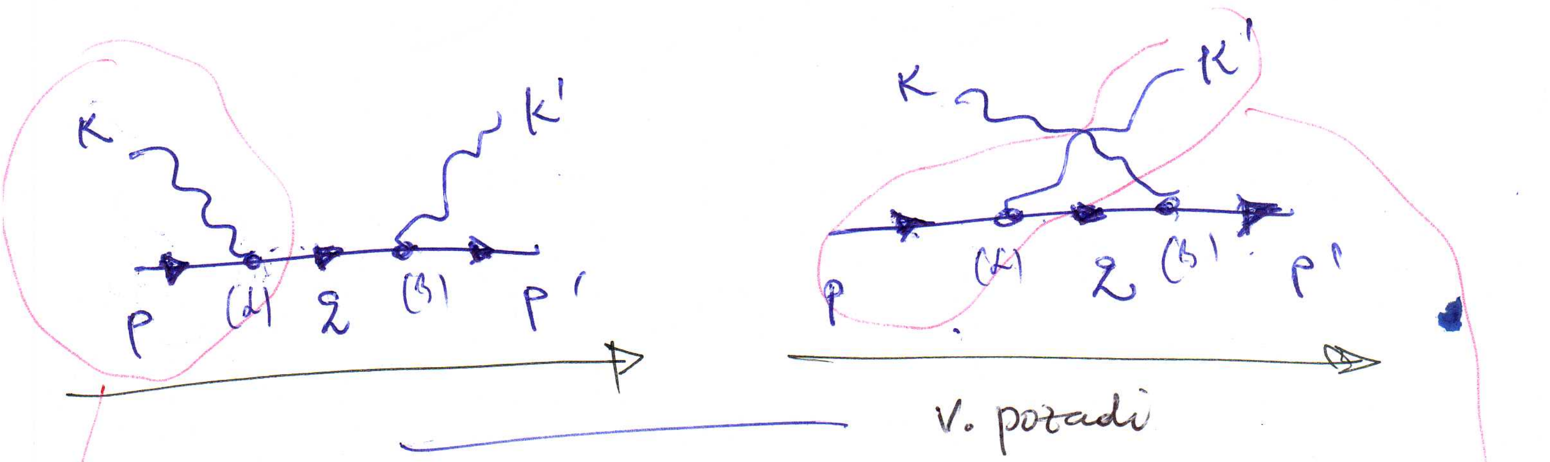
c) $\mathcal{M} = ie \quad \bar{v}(\vec{q})\, \not{\epsilon}(\vec{k}')\, u(\vec{p})$



d) $\mathcal{M} = ie \ \bar{u}(\vec{q}')\, \not{\epsilon}(\vec{k}')\, v(\vec{p}')$



Kako glase Fajnmanove amplitude za Komptonovo rasejanje na elektronu? Odgovarajući Fajnmanovi dijagrami u drugom redu popravke su dati kao:



a) $$\mathcal{M}_a = \bar{u}(\vec{p}\,')\, ie\gamma^{\beta}\varepsilon_{\beta}(\vec{k}\,')\; i S_F(q)\; ie\gamma^{\alpha}\varepsilon_{\alpha}(\vec{k})\, u(\vec{p})$$

$$k+p=q$$

$$\mathcal{M}_a = -e^2\, \bar{u}(\vec{p}\,')\, \not{\varepsilon}(\vec{k}\,')\; i S_F(k+p)\; \not{\varepsilon}(\vec{k})\, u(\vec{p})$$

b) $$\mathcal{M}_b = -e^2\, \bar{u}(\vec{p}\,')\, \not{\varepsilon}(\vec{k})\; i S_F(q)\; \not{\varepsilon}(\vec{k}\,')\, u(\vec{p})$$

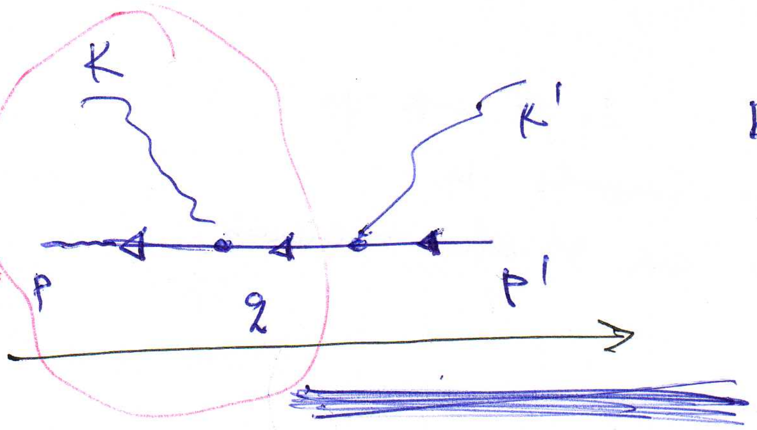
$$p = k'+q$$

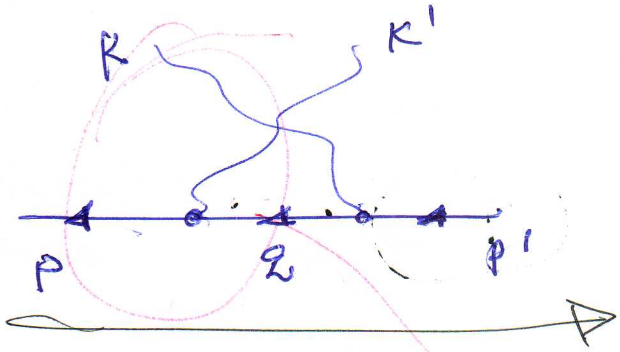
$$q = p-k'$$

Ukupna amplituda
$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b$$

$$\mathcal{M}_b = -e^2\, \bar{u}(\vec{p}\,')\, \not{\varepsilon}(\vec{k})\; i S_F(p-k')\; \not{\varepsilon}(\vec{k}\,')\, u(\vec{p})$$

4. Kako glase Fajnmanove amplitude za Komptonovo rasejanje na pozitronu? Odgovarajući Fajnmanovi dijagrami u drugom redu popravke su dati kao:

a) 

b) 

a)

$$\mathcal{M}_a = e^2\, \bar{v}_r(\vec{p})\, \not{\epsilon}_r(\vec{k})\, i S_F(q)\, \not{\epsilon}_r(\vec{k}')\, v_r(\vec{p}\,')$$

Primetiti: nema minusa - zbog "smera impulsa" u dijagramu

$q = -p-k$ pa je $\mathcal{M}_a = e^2\, \bar{v}_r(\vec{p})\, \not{\epsilon}_r(\vec{k})\, i S_F(-p-k)\, \not{\epsilon}_r(\vec{k}')\, v_r(\vec{p}\,')$

v. pozadi

b)

$$\mathcal{M}_b = e^2\, \bar{v}_r(\vec{p})\, \not{\epsilon}_r(\vec{k}')\, i S_F(q)\, \not{\epsilon}_r(\vec{k})\, v_r(\vec{p}\,')$$

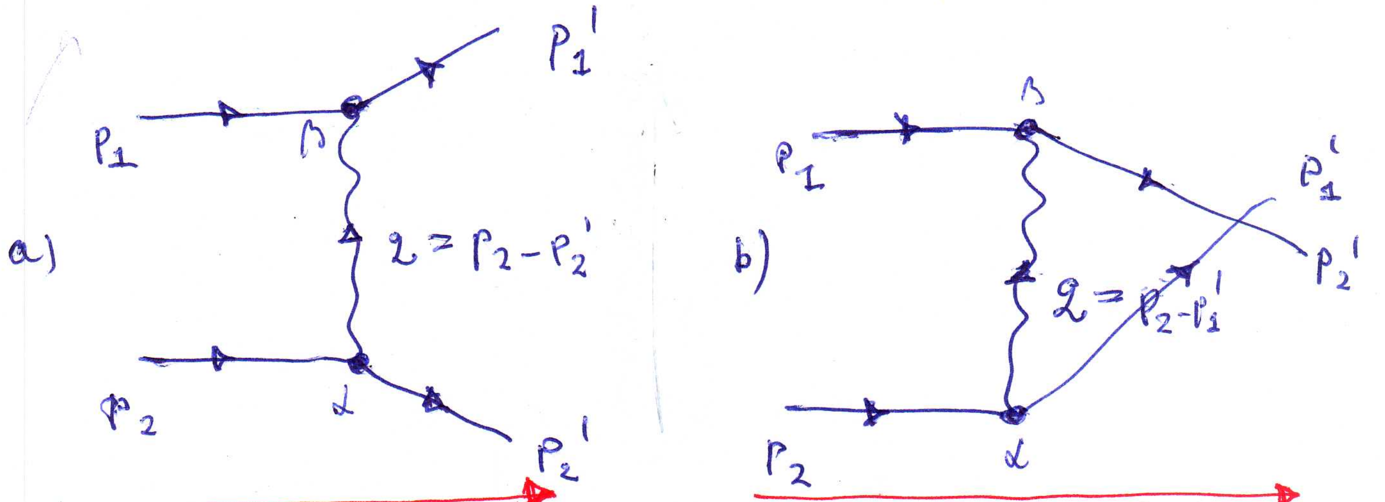
$q = -p+k'$

v. pozadi

$$\boxed{\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b}$$

$$\mathcal{M}_b = e^2\, \bar{v}_r(\vec{p})\, \not{\epsilon}_r(\vec{k}')\, i S_F(-p+k')\, \not{\epsilon}_r(\vec{k})\, v_r(\vec{p}\,')$$

Kako glase Fajnmanove amplitude za elektronsko-elektronsko rasejanje? Fajnmanovi dijagrami u drugom redu popravke su:



Møller scattering
Danish physicist

a)

$$\mathcal{M}_1 = \bar{u}(\vec{p}_1{}') ie\gamma^\beta u(\vec{p}_1)\; iD_{F\beta\alpha}(q)\, \bar{u}(\vec{p}_2{}') ie\gamma^\alpha u(\vec{p}_2)$$

$$= -e^2 \underbrace{\bar{u}(\vec{p}_1{}')\gamma^\beta u(\vec{p}_1)}\; \underbrace{iD_{F\beta\alpha}(p_2 - p_2')}\, \underbrace{\bar{u}(\vec{p}_2{}')\gamma^\alpha u(\vec{p}_2)}$$

b) Ova amplituda se dobija zamenom

$\vec{p}_2{}' \to \vec{p}_1{}'$ i $\vec{p}_1{}' \to \vec{p}_2{}'$: antisimetrizacija, Paulijev princip ali sa suprotnim znakom

$$\mathcal{M}_2 = +e^2\, \bar{u}(\vec{p}_2{}')\gamma^\beta u(\vec{p}_1)\; iD_{F\beta\alpha}(p_2 - p_1')\,\bar{u}(\vec{p}_1{}')\gamma^\alpha u(\vec{p}_2)$$

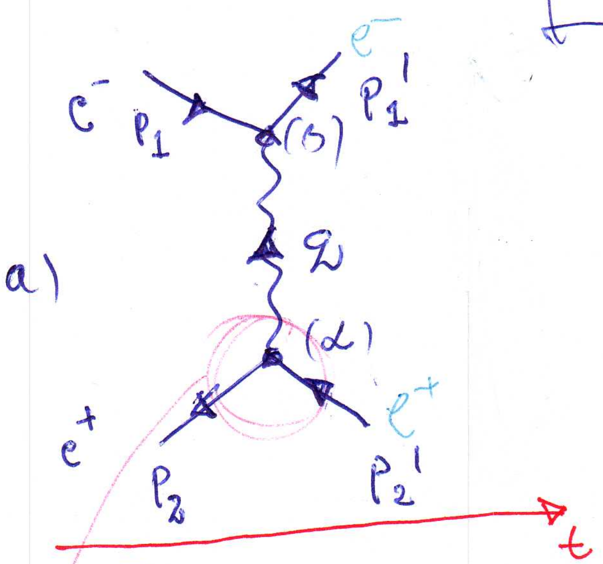
Ukupna amplituda

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 \longrightarrow$$

6. Kako glase Fajnmanove amplitude za elektronsko – pozitronsko rasejanje? Fajnmanovi dijagrami u drugom redu popravke su dati kao:

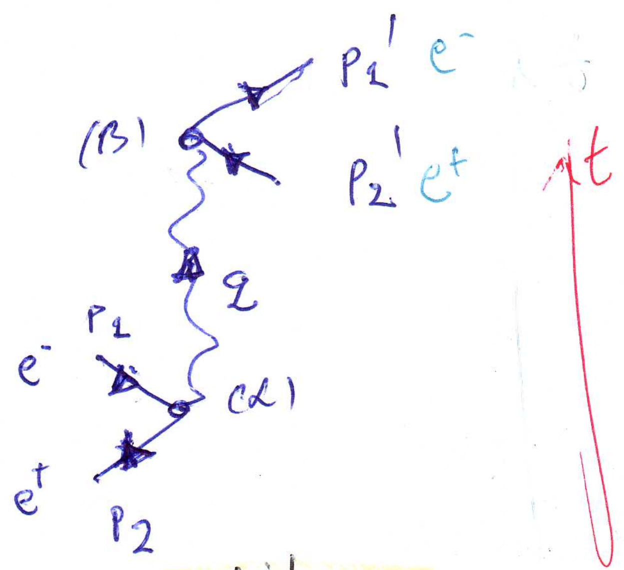
$\boxed{\text{Bhabha}}$

a)



Rasejanje

b)



Anihilacija

$$\mathcal{M}_a = -e^2 \bar{u}(\vec{p}_1{}')\gamma^\beta u(\vec{p}_1)\, i D_{F\beta\alpha}(q)\, \bar{v}(\vec{p}_2)\gamma^\alpha v(\vec{p}_2{}')$$

$$q = p_2' - p_2 \quad \checkmark$$

$$\mathcal{M}_b = +e^2 \bar{u}(\vec{p}_1{}')\gamma^\beta v(\vec{p}_2{}')\, i D_{F\beta\alpha}(q)\, \bar{v}(\vec{p}_2)\gamma^\alpha u(\vec{p}_1)$$

Zbog pravila 8

Naravno, $\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b$

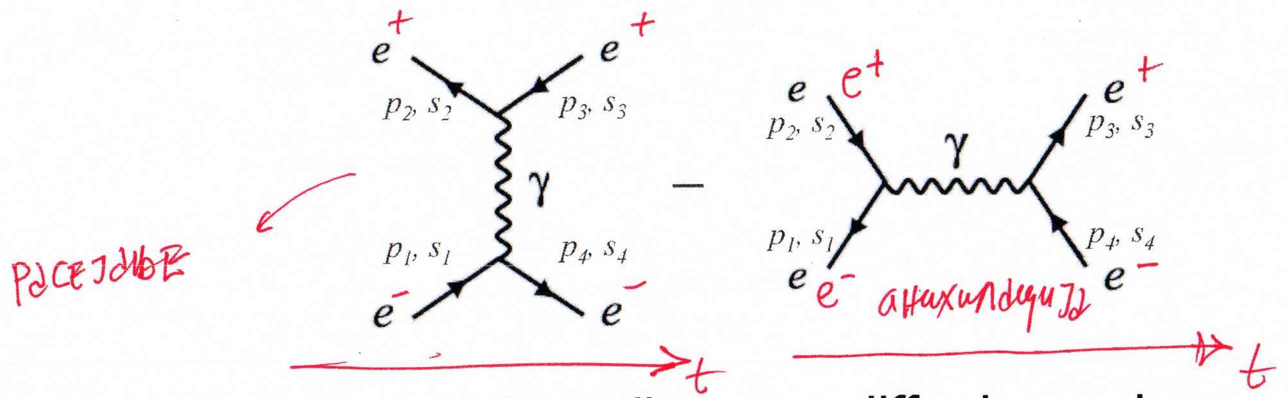
Čvor u a) tiče se pozitrona

$$-p_2' + q = -p_2 \Rightarrow q = p_2' - p_2$$

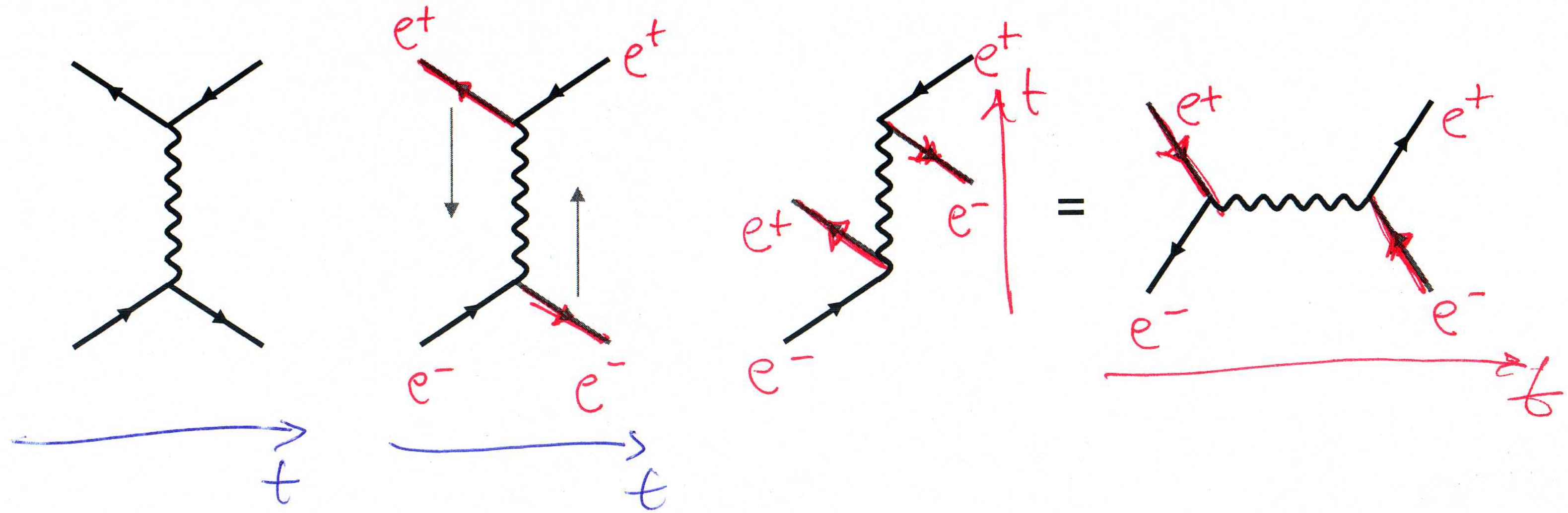
Čvor u b) $\quad p_1 - p_2 = q$ (pozitron)

More on anti-symmetrization of QED diagrams

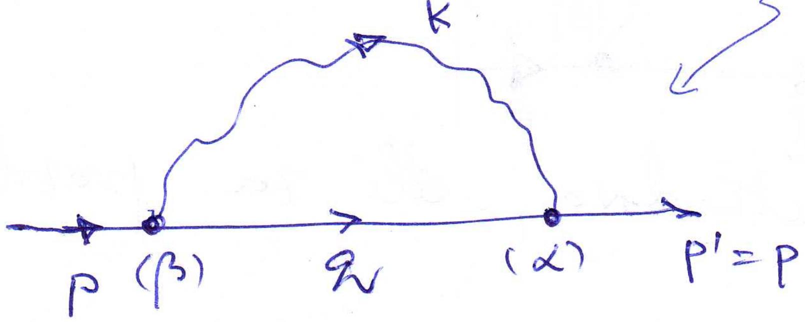
Consider electron positron scattering: $e^+e^- \rightarrow e^+e^-$ (Bhabha scattering)



Relative negative sign since diagrams differ by exchange of incoming positron and outgoing electron:



1. Dat je Fajnmanov dijagram. Napisati Fajnmanovu amplitudu!



Lepton okružen virtuelnim fotonima

$$\mathcal{M} = \frac{-e^2}{(2\pi)^4}\int d^4k\, i D_{F\alpha\beta}(k)\, \bar{u}(\vec{p})\gamma^\alpha\, i S_F(p-k)\gamma^\beta u$$

Primetiti $\quad p = q + k = p'$

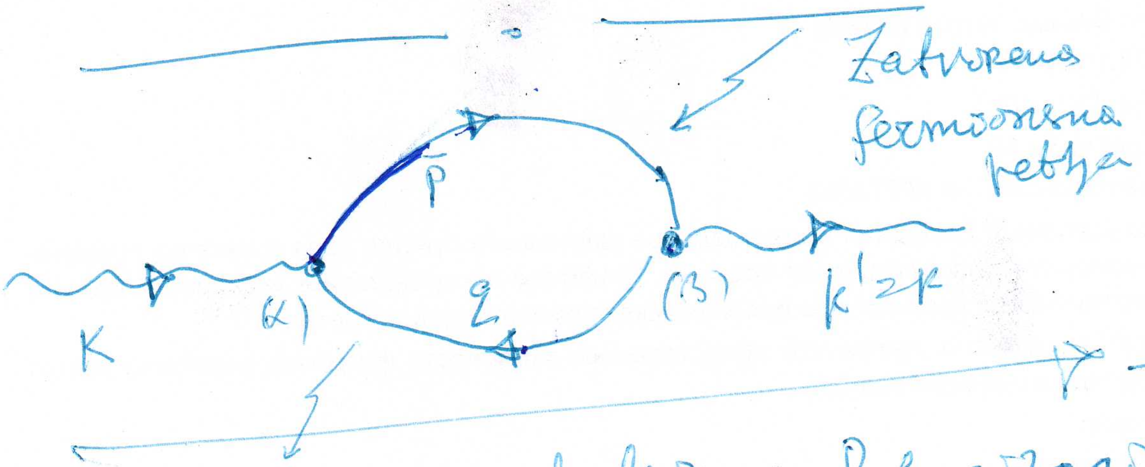
q i k nisu određeni $\rightarrow$ integracija $\frac{1}{(2\pi)^4}\int d^4k$

Gore je integracija po k, a može i po q sve je jedno!

$$\mathcal{M} = \frac{-e^2}{(2\pi)^4}\int d^4q\, i D_{F\alpha\beta}(p-q)\, \bar{u}(\vec{p})\gamma^\alpha\, i S_F(q)\gamma^\beta u(\vec{p})$$

Domaći $\longrightarrow$

8. Dat je Fajnmanov dijagram. Kako glasi Fajnmanova amplituda?



Foton okružen virtuelnim ~~fotonima~~ FERMIONIMA : Polarizacija vakuuma

Pravilo 6: Za svaku fermionsku petlju uzima se trag izraza i množi sa (−1)

$$K = -p + q \implies q = K + p$$

$$\sim \left[\gamma^{\alpha} \varepsilon_{r\alpha}(\vec{k})\, i S_F(q)\, \gamma^{\beta} \varepsilon^{*}_{r\beta}(\vec{k})\, i S_F(p) \right]$$

$$\left[\not{\varepsilon}_r(\vec{k})\, i S_F(K+p)\, \not{\varepsilon}_r(\vec{k})\, i S_F(p) \right]$$

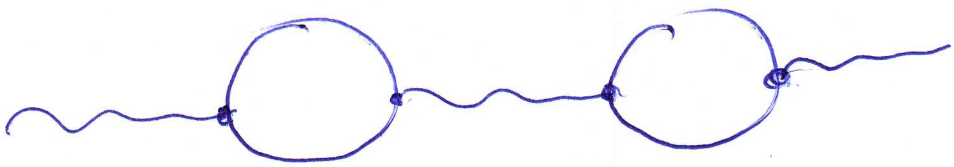
$$\boxed{p \ \text{neodređeno}}$$

$$\mathcal{M} = (-1) \frac{(-e^2)}{(2\pi)^4} \int d^4p \, \mathrm{Tr}\left[\not{\varepsilon}_r(\vec{k})\, i S_F(K+p)\, \not{\varepsilon}_r(\vec{k})\, i S_F(p) \right]$$

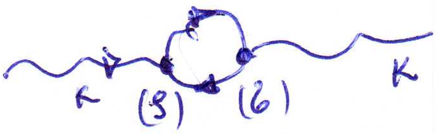
$$\Downarrow$$

$$\mathcal{M} = \mathcal{M}(K) \ !$$

1. Za dati Fajnmanov dijagram, naći Fajnmanovu amplitudu:

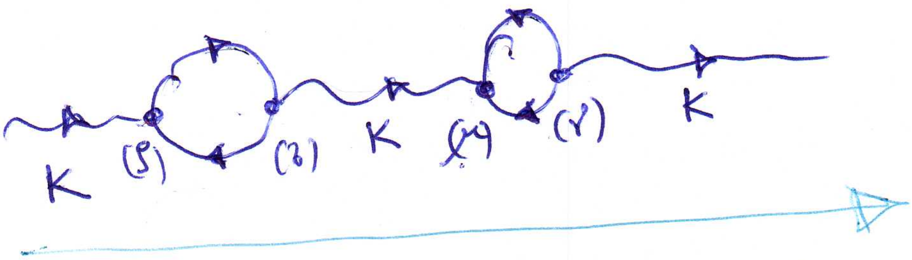


U jednom od prethodnih zadataka nađena je amplituda za dijagram



$$\equiv \mathcal{M}_1^{\beta\delta}(k)$$

Sadašnji dijagram



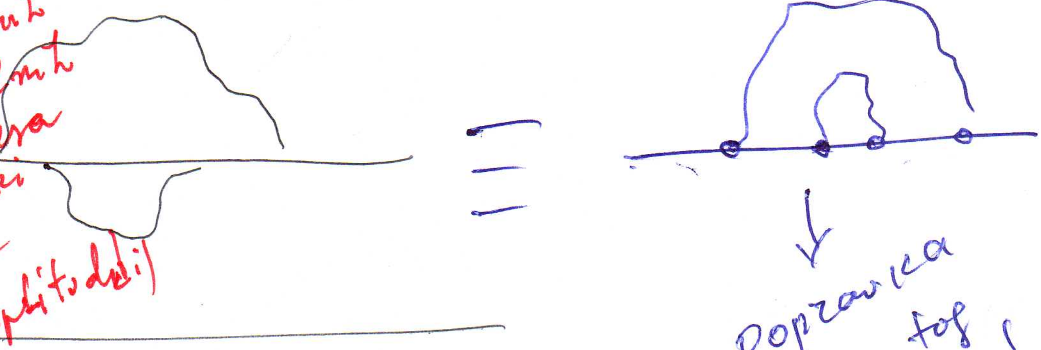
$$\equiv \mathcal{M}$$

$$\mathcal{M} = \mathcal{M}_1^{\beta\delta}(k)\; i D_{F\delta\mu}(k)\; \mathcal{M}_2^{\mu\nu}(k)$$

Popravka 4-tog reda!

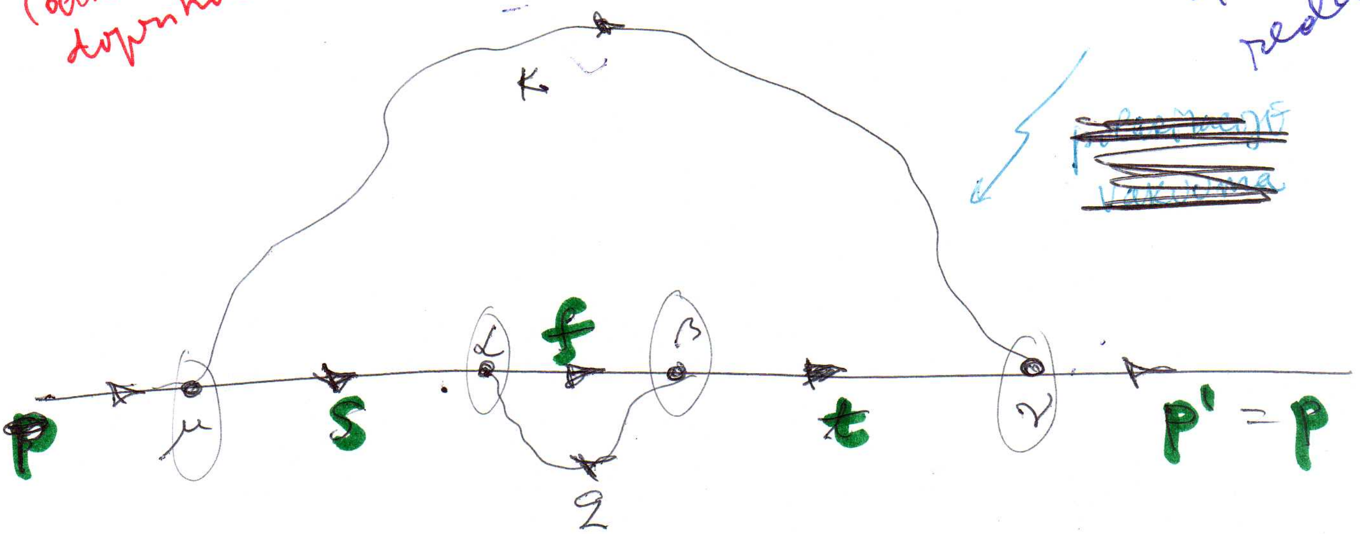
Dat je Fajnmanov dijagram na slici. Kako glasi odgovarajuća Fajnmanova amplituda

Napomenuti, da za razliku od realnih fotona, kod virtuelnih fotona od ~~interera~~ su sva 4 polarizaciona vektora (odnosno, sva 4 amplitude) doprinose



Popravka 4-tog reda!

~~polarizacija vakuuma~~



Prebrojati: 5 propagatora!

μ verteks $\quad p-k-s=0 \Rightarrow s=p-k$

α verteks $\quad s-f-q=0 \Rightarrow f=s-q=p-k-q$

β verteks $\quad f+q-t=0 \Rightarrow t=f+q=p-k-q+q = p-k$

$$\mathcal{M} \sim \bar{u}(\vec{p}')\, ie\gamma^\nu\, i S_{F\nu\beta}(t)\, ie\gamma^\beta\, i S_{F\beta\alpha}(f)\, ie\gamma^\alpha\, i D_{F\beta\alpha}(q)\, i S_{F\alpha\mu}(s) \cdot$$

$$\cdot\, ie\gamma^\mu\, i D_{F\nu\mu}(k)\, u(\vec{p})$$

$i^5 = i$, $(ie)^4$

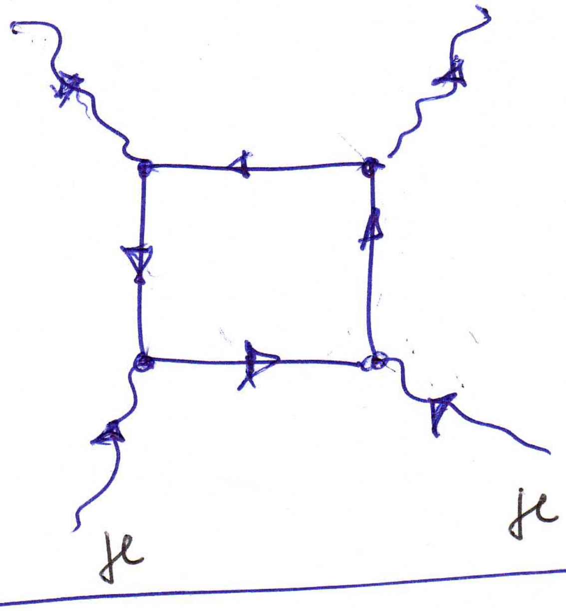
$$\mathcal{M} = i(ie)^4 \int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\, \bar{u}(\vec{p})\, \gamma^{\gamma} S_{F\gamma\beta}(p-k)\, \gamma^{\beta} S_{F\beta\alpha}(p-k-q)\,\cdot$$

$$\gamma^{\alpha} D_{F\beta\alpha}(q)\, S_{F\alpha\mu}(p-k)\, \gamma^{\mu} D_{F\gamma\mu}(k)\, u(\vec{p})$$

Domaći: Uraditi isto za pozitron!

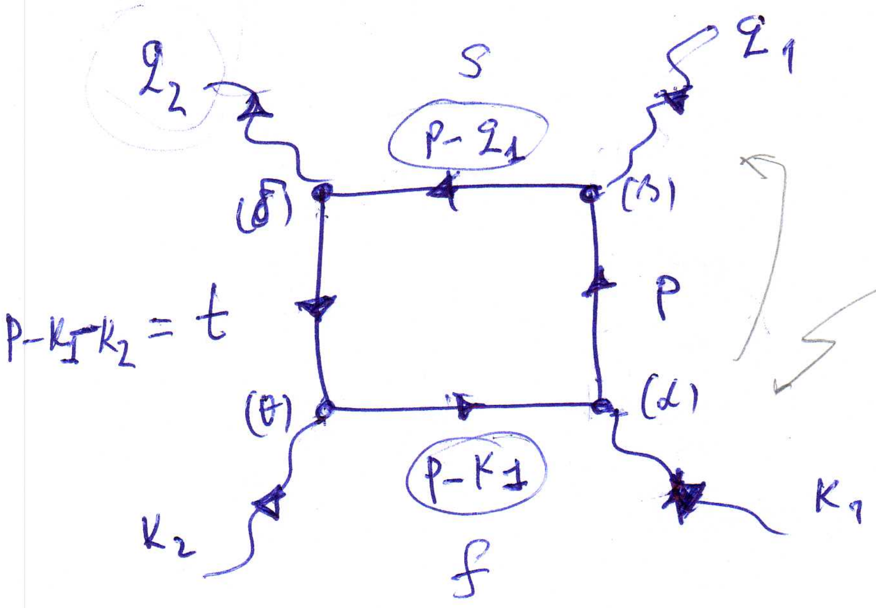
$$\mathcal{M} = \mathcal{M}(p)$$

1) Za dati Fajnmanov dijagram, napisati Fajnmanovu amplitudu:



$\longrightarrow$ (korekcija iz četvrtog reda perturbacije)

Rasejanje fotona



Kreni od ovog čvora.

$p \longrightarrow$ neodređeno

$(\beta) \rightarrow \quad -s+p-q_1=0 \Rightarrow \quad s = p-q_1$

$(\alpha) \rightarrow \quad f+k_1-p=0 \Rightarrow f = p-k_1$

$(\theta) \rightarrow \quad t+k_2-f=0 \Rightarrow t = f-k_2$

$= p-k_1-k_2$

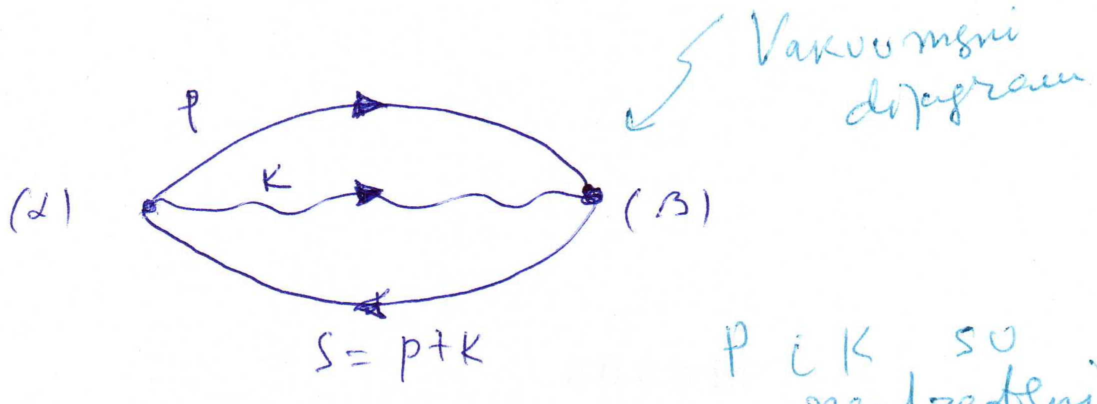
/ bez kose crte!

$$(-1)\int\frac{d^4p}{(2\pi)^4}\, tr\Big[i S_{F\alpha\theta}(f)\, ie\gamma^{\theta}\, \cancel{\epsilon}_{\theta}(\vec{k}_2)\, i S_{F\theta\delta}(t)\, ie\gamma^{\delta}\epsilon_{\delta}(\vec{q}_2)\, i S_{F\delta\beta}(s)\, ie\gamma^{\beta}\epsilon_{\beta}(\vec{\epsilon}_1)\, i S_{F\beta\alpha}(p)\, ie\gamma^{\alpha}\epsilon_{\alpha}(\vec{k}_1)\Big] = \mathcal{M}$$ odnosno

$$(-1)\,(ie)^4\,(i)^4\int\frac{d^4p}{(2\pi)^4}\, tr\Big[S_{F\alpha\theta}(p-k_1)\,\cancel{\epsilon}(\vec{k}_2)\,S_{F\theta\delta}(p-k_1-k_2)\,\cancel{\epsilon}(\vec{q}_2)\,S_{F\delta\beta}(p-q_1)\,\cancel{\epsilon}(q_1)\,S_{F\beta\alpha}(p)\,\cancel{\epsilon}(\vec{k}_1)\Big] = \mathcal{M}$$

$$\mathcal{M} = \frac{(ie)^4}{(2\pi)^4}\int d^4p\, tr\Big[S_{F\alpha\theta}(p-k_1)\,\cancel{\epsilon}(\vec{k}_2)\,S_{F\theta\delta}(p-k_1-k_2)\,\cancel{\epsilon}(\vec{q}_2)\,S_{F\delta\beta}(p-q_1)\,\cancel{\epsilon}(q_1)\,S_{F\beta\alpha}(p)\,\epsilon(\vec{k}_1)\Big]$$

2. Za dati Fajnmanov dijagram, napisati Fajnmanovu amplitudu:



p i k su neodređeni

$$\mathcal{M} \sim \left[i S_F(k+p)\,(ie\gamma^\beta)\, i D_{F\beta\alpha}(k)\, i S_F(p)\,(ie\gamma^\alpha) \right]$$

$$\sim i S_F(k+p)\,(ie\gamma^\beta)\, i S_F(p)\,(ie\gamma^\alpha)\, i D_{F\beta\alpha}(k)$$

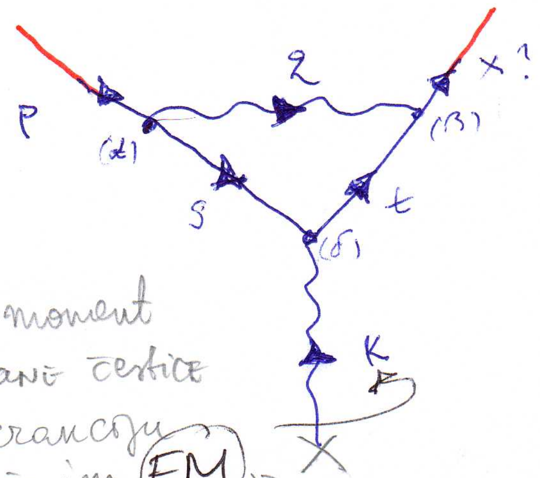
$$\mathcal{M} = (-1) \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left[\underline{i} S_F(k+p)(\underset{\sim}{i}e\gamma^\beta)\, \underline{i} S_F(p)\, \underset{\sim}{i}e\gamma^\alpha \right] \underline{i}\, D_{F\beta\alpha}(k)$$

$$\mathcal{M} = i(ie)^2 \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left[S_F(k+p)\,\gamma^\beta S_F(p)\gamma^\alpha \right]$$

$$D_{F\beta\alpha}(k)$$

Ovakvi dijagrami mogu biti ispušteni jer ne doprinose proračunu!

Za dati Fajnmanov dijagram, naći Fajnmanovu amplitudu:



Magnetni moment naelektrisane čestice kroz interakciju sa spoljašnjim EM poljem (statično polje)

→ Dijagram u okviru računanja magnetnog momenta elektrona

(α): $p-q-s=0 \Rightarrow s=p-q$

(δ): $s+k-t=0 \Rightarrow t=s+k=p-q+k$

(β): $q+t-x=0 \Rightarrow x=q+t \Rightarrow x=p-q+k+q=p+k$

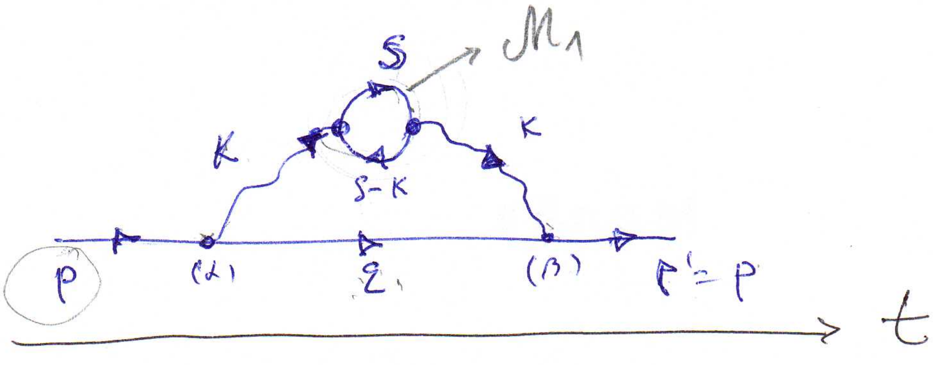
$$\boxed{q - \text{neodređeno}}$$

$$\mathcal{M}=\int\frac{d^4q}{(2\pi)^4}\,\bar{u}(\vec{p}+\vec{k})\,(ie\gamma^{\beta})\,iS_F(t)\,(ie\gamma^{\delta})\,\epsilon_{\delta}(\vec{k})\,iS_F(s)\,(ie\gamma^{\alpha})\,u(\vec{p})\,\boxed{iD_{F\beta\alpha}(q)}$$

Фотон из спољашњег ЕМ поља

$$=-i\,(ie)^3\int\frac{d^4q}{(2\pi)^4}\,\bar{u}(\vec{p}+\vec{k})\,\gamma^{\beta}S_F(p-q+k)\,\gamma^{\delta}\epsilon_{\delta}(\vec{k})\,S_F(p-q)\,\gamma^{\alpha}\,u(\vec{p})\,D_{F\beta\alpha}(q)$$

4. Za dati Fajnmanov dijagram, napisati Fajnmanovu amplitudu:



K i q neodređeni

$$\mathcal{M} \sim \bar{u}(\vec{p})\,(ie\gamma^{\beta})\, i S_F(q)\,(ie\gamma^{\alpha})\, u(\vec{p})\, \mathcal{M}_1(K)$$

$$\mathcal{M}_1(K) = (-1)(ie)^2 \int \frac{d^4 s}{(2\pi)^4}\, tr\left[\not{\epsilon}^*(\vec{K})\, i S_F(s-K)\, i S_F(s)\, \not{\epsilon}(\vec{K})\right]$$

(α): $p - q - K = 0 \Rightarrow q = p - K$

$$\mathcal{M} : i(ie)^2 \int \frac{d^4 K}{(2\pi)^4}\, \bar{u}(\vec{p})\,\gamma^{\beta} S_F(p-K)\,\gamma^{\alpha} u(\vec{p})\,.$$

$$\underbrace{(-1)(ie)^2 \int \frac{d^4 s}{(2\pi)^4}\, tr\left[\not{\epsilon}^*(\vec{K})\, S_F(s-K)\, S_F(s)\,\not{\epsilon}(\vec{K})\right]}_{\mathcal{M}_1(K)}$$

Konačno:

$$\mathcal{M} = -i(ie)^4 \int \frac{d^4 K}{(2\pi)^4}\,\frac{d^4 s}{(2\pi)^4}\, \bar{u}(\vec{p})\,\gamma^{\beta} S_F(p-K)\,\gamma^{\alpha} u(\vec{p})$$

$$tr\left[\not{\epsilon}^*(\vec{K})\, S_F(s-K) S_F(s)\,\not{\epsilon}(\vec{K})\right] = \mathcal{M}(p)$$